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Overall degradation of conductive solids with mesocracks

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Abstract—A continuum damage model for heat conduction is proposed in this work to account for the degradation of the bulk thermal conductivity due to the formation of subscale defects in the solid. Specifically, defects in the form of cracks, or mesocracks, have been considered. A field theory is developed analytically which isolates the effects of mesocracks in an added matrix of the thermal resistance. Followed by a general formulation, the concept of mesocrack damage in the process of heat transport is further illustrated by considering a three-dimensional and isotropic medium transferring the thermal energy according to Fourier's law of heat conduction. It is found that the amount of degradation of the bulk thermal conductivity increases linearly with the mesocrack density. In the case that the solid is saturated with mesocracks, a limiting value of thermal conductivity $\bar{k}$ being one-ninth of the intact value is obtained. A detailed discussion on this limiting value is then provided with emphasis placed on the identification of the damage parameters in the temperature-gradient model proposed earlier for assessing the mesocrack damage from a different point of view.

INTRODUCTION

MANY materials such as rock, concrete, ceramic, etc. have existing crack structures generated either from natural consequences or from manufacturing processes. The size of the continuum elements for these materials should be large enough to include sufficient populations of these subscale cracks such that it adequately reflects average bulk material behavior. In metallic materials, the sizes of subscale structures such as voids, grains, dislocations, etc. are measured in the ångström to micron range and are much smaller than that for the macroscopic scale measured in the centimeter or up level. Thus, these subscale structures are referred to as microscale structures. The subscale cracks in rock, concrete, etc. are in the size range from 100 μm to millimeters. Consequently, the size of these cracks may be referred to as *mesoscale* and the cracks *mesocracks*. When geometrical defects are initiated in the processes of momentum or heat transfer in a solid medium, the overall load- or energy-carrying capacity of the solid usually degrades due to the creation of new free surfaces. If the characteristic dimensions of the defects are in the same order as the global dimensions of the solid, the effects of the defects can be resolved via a macroscopic approach by specifying the boundary conditions at the surface(s) of the defect. Typical examples include the temperature fluctuations ahead of a slowly propagating crack [1] and the thermal field around a macrocrack tip [2] in the solid. Generally speaking, a macrocrack with sharp tips induces a $1/\sqrt{r}$-type singularity for the temperature gradient, with r being the distance measured from the crack tip, while the temperature remains bounded as the crack tip is approached.

In studying the influences of the mesocracks on the energy-carrying capacity of the solid medium, it is worthwhile to make a qualitative assessment on the size effects of a crack on the singularity of the near-tip temperature gradient. The temperature and the temperature gradient fields obtained by Sih [2] in the vicinity of a macrocrack tip can be expressed by

$$\begin{aligned} T(r,\theta) &= -2H_0(ar)^{1/2}\sin(\theta/2) \\ T_{,r}(r,\theta) &= -H_0(a/r)^{1/2}\sin(\theta/2) \\ \text{with } H_0 &= T_{,y}^{\infty}/\sqrt{2} \end{aligned} \tag{1}$$

where θ denotes the polar angle measured from the leading edge of the crack tip, a the half length of the crack, $T_{,y}^{\infty}$ the remote temperature gradient applied to the cracked solid, and the subscript denotes the partial differentiation with respect to the corresponding coordinates. The quantity $\sqrt{a}H_0$ is called the strength of the singularity of the temperature gradient which depends on the remote temperature gradient and the size of the crack. For a macrocrack with a being of the order of 1 m, for example, equation (1) depicts a 'near-tip' temperature gradient at $r \simeq 0.1$ mm of approximately 70 times higher than that applied

NOMENCLATURE

a	characteristic dimension of the mesocrack [m]
e	unit normal vectors along the coordinate axes
f	volume fraction of the mesocracks
g	temperature function per unit heat flux [$m^2 K W^{-1}$]
G	Green's function of the temperature per unit heat flux [$K W^{-1}$]
H	added tensor in the overall thermal resistance tensor [$m K W^{-1}$]
k	thermal conductivity [$W m^{-1} K^{-1}$]
n	unit normal vector of the crack surface
N	total number of cracks per unit volume
Q	heat flux applied perpendicularly to the boundary surface [$W m^{-2}$]
r	radial distance measured along the crack surface from the center of the crack [m]
S	boundary surface of the entire body [m^2]
T	temperature [K]
V	total volume of the entire body [m^3]
x	spatial coordinates [m]
y	coordinate perpendicular to a line crack.

Greek symbols

β	transformation matrix between the prime and the physical coordinate systems
δ	Kronecker delta function
θ	Euler angle [deg]
ξ	material point at the boundary surface of the entire body
ϕ	Euler angle [deg].

Subscripts and superscripts

X_i	component of vector X in the x_i-direction, $i = 1, 2, 3$
X_R	ratio of the quantity X of the matrix to that of the air
X_{ij}	indicial notation for the tensorial quantity X
$X_{,y}$	$\partial X / \partial y$
$X_{,i}$	$\partial X / \partial x_i$
X^C	physical quantity X in the cavity
X^M	physical quantity X in the matrix
$\bar{X}$	volumetric average of X
X'	prime coordinates aligned to the cavity
X^{-1}	inverse of X.

remotely and the singularity of the temperature gradient is quite obvious. For a small crack with characteristic dimension $a \simeq 0.1$ mm, a material point at the same distance of $r \simeq 0.1$ mm only experiences a temperature gradient of 0.7 times of $T^{\infty}_{,y}$. In other words, when the size of the crack becomes small, the strength of the singularity of the temperature gradient no longer serves as a reliable index for measuring the intensified energy concentrated at the crack tip and a different measure has to be introduced. In reality, because we cannot afford to simulate hundreds of mesocracks on a one-by-one basis, the mesocracking effects are most appropriately described by a continuum theory which provides a post-damage assessment on the degradation of the energy carrying capacity of the solid in an overall or bulk sense.

The discipline of continuum damage mechanics has been developed for this purpose. In essence, it investigates the evolution of the constitutive behavior of the solid resulting from the loss of the granular integrity in load transmission. The research in this field is still ongoing and the main effort is devoted to the cumulative degradation of the mechanical properties of the material from their intact values. Detailed discussions and the related publications on this subject can be found in an excellent review paper by Krajcinovic [3] and a newly published book by Kachanov [4]. In these articles, the damage concept is discussed for ductile materials with intrinsic non-linear stress–strain curves and under cyclic loads. In assessing the quasi-brittle material degradation, Budiansky and O'Connell [5] first derived expressions for the degraded material properties due to mesocracking. Later in 1983, Horii and Nemat-Nasser [6] extended these expressions to account for the effects of mechanical anisotropy due to the closing of the crack surfaces. Chen [7], Kipp *et al.* [8]. and Taylor *et al.* [9] developed dynamic continuum damage theories which included explicitly the strain rate effects. This model was recently extended [10, 11] in studying the cumulative material damage under quasi-static loading conditions. Generally speaking, a direct consequence of material damage is the non-linear behavior of the stress vs the strain response. This is the direct result of the evolutionary characteristic of damage and its interaction with applied loads.

While the continuum damage mechanics on the load-bearing capacity of solids is developing, the effects of the material damage on the loss of energy-carrying capacity of the solid has not yet attracted sufficient attention. In a recent work [12], an attempt was made to accommodate the degradation of the bulk thermal conductivity in a solid medium due to mesocrack formation in the thermal loading history. This work is essentially an extension of the model proposed earlier in refs. [10, 11]. The establishment of a common damage measure for evaluating the degradation of the energy- and the momentum-carrying capacities facilitates the consideration of thermal/mechanical interactions as an entirety in the history of damage evolution. The evolution of the damage was discussed in great detail in ref. [12].

As a continuation of the development, the present study aims to derive expressions to describe the overall degradation of the bulk thermal conductivity of a material volume due to the presence of mesocracks. Previous work by Hoenig [13] and Hasselman [14] dealt with the same subject. The present approach is mathematically rigorous and is in parallel to that suggested by Horii and Nemat-Nasser [6] in continuum damage mechanics which investigates the overall degradation of elastic moduli via the concept of the volume fraction of the microcracks in a Hookean matrix. Finally, the way in which damage evolution can be coupled into these expressions has been suggested.

WEAKENING OF A FOURIER SOLID BY MESOCRACKS

The influences of the mesocracks on the energy carrying capacity of a solid medium can be analyzed by considering a three-dimensional solid with randomly distributed mesocracks as shown in Fig. 1. The solid has a total volume of V and is subject to a heat flux vector Q perpendicular to its boundary surface S. In the sequel the physical and the geometrical quantities with superscripts C and M denote, respectively, those for the mesocracks and the matrix material, and

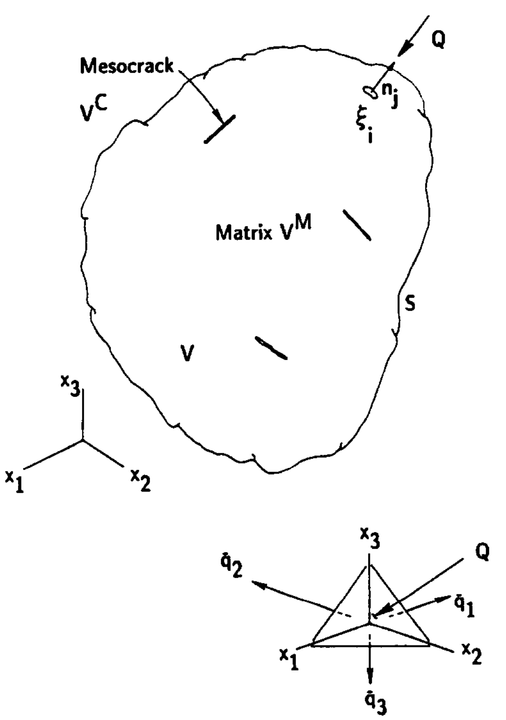

FIG. 1. A three-dimensional medium containing randomly oriented mesocracks.

the full quantities without a superscript denote those for the entire body. Obviously

$$V = V^{\mathrm{C}} + V^{\mathrm{M}}. \tag{2}$$

Based on the volume average concept, the average temperature $\bar{T}$, the average temperature gradient $\bar{T}_{,i}$, and the average heat flux $\bar{q}_i$ over the entire body can be expressed as

$$[\bar{T}, \bar{T}_{,i}, \bar{q}_i] = \frac{1}{V}\int_V [T(x_j), T_{,i}(x_j), q_i(x_j)]\,\mathrm{d}V \tag{3}$$

with x_i being the spatial coordinates of a material point and $i, j = 1, 2$ and 3 in the Cartesian coordinate system. Similarly, the average quantities for the mesocracks and the matrix material can be written in the following form:

$$[\bar{T}^{\mathrm{C}}, \bar{T}^{\mathrm{C}}_{,i}, \bar{q}^{\mathrm{C}}_i] = \frac{1}{V^{\mathrm{C}}}\int_{V^{\mathrm{C}}} [T(x_j), T_{,i}(x_j), q_i(x_j)]\,\mathrm{d}V$$

for the mesocracks and

$$[\bar{T}^{\mathrm{M}}, \bar{T}^{\mathrm{M}}_{,i}, \bar{q}^{\mathrm{M}}_i] = \frac{1}{V^{\mathrm{M}}}\int_{V^{\mathrm{M}}} [T(x_j), T_{,i}(x_j), q_i(x_j)]\,\mathrm{d}V \tag{4}$$

for the matrix material. The average quantities in the mesocracks and the matrix are related to those of the entire body by the volume fraction rule, i.e.

$$\begin{aligned} \bar{q}_i &= (1-f)\bar{q}^{\mathrm{M}}_i + f\bar{q}^{\mathrm{C}}_i \\ \bar{T}_{,i} &= (1-f)\bar{T}^{\mathrm{M}}_{,i} + f\bar{T}^{\mathrm{C}}_{,i}, \text{ etc.} \end{aligned} \tag{5}$$

where f is the volume fraction of the mesocracks defined as V^{C}/V. Let us now focus our attention on the average temperature gradient $\bar{T}_{,i}$ in equation (5). If the heat transport process in the matrix material is assumed to be homogeneous and following Fourier's law of heat conduction

$$\bar{q}^{\mathrm{M}}_i = k_{ij}\bar{T}^{\mathrm{M}}_{,j} \quad \text{in } V^{\mathrm{M}} \tag{6}$$

and consequently

$$\bar{T}^{\mathrm{M}}_{,i} = R_{ij}\bar{q}^{\mathrm{M}}_j \quad \text{in } V^{\mathrm{M}} \tag{7}$$

where R_{ij} denotes the inverse of the conductivity matrix, k_{ij}^{-1}, of the matrix material. Substituting the expression for $\bar{q}^{\mathrm{M}}_i$ from equation (5) into equation (7), and the result for $\bar{T}^{\mathrm{M}}_{,i}$ into the second of equation (5), we obtain the expression for the overall temperature gradient averaged over the entire body

$$\bar{T}_{,i} = R_{ij}\bar{q}_j - fR_{ij}\bar{q}^{\mathrm{C}}_j + f\bar{T}^{\mathrm{C}}_{,i} \tag{8}$$

where the complicated heat transfer modes in the aerial closure between the mesocrack surfaces are included in the term $\bar{q}^{\mathrm{C}}_j$. The last term containing the temperature gradient in the mesocracks

$$f\bar{T}^{\mathrm{C}}_{,i} = \frac{1}{V}\int_{V^{\mathrm{C}}} T_{,i}\,\mathrm{d}V \tag{9}$$

can be related to the average heat flux vector $\bar{q}_j$ of the entire body according to the divergence theorem. We first relate the temperature gradient averaged over the total volume of the mesocracks to the temperature specified over the mesocrack surfaces:

$$f\bar{T}^C_{,i} = \frac{1}{V}\int_{S^C} Tn_i \,\mathrm{d}S^C. \quad (10)$$

Then, by denoting the Green's function $G(x_i, \xi_i)$ for the temperature induced at the material point x_i by a unit heat flux applied at ξ_i on S, the temperature $T(x_i)$ in equation (10) can be expressed by

$$T(x_i) = \int_S G(x_i, \xi_i)Q(\xi_i)\,\mathrm{d}S, \quad \text{for } \xi_i \in S. \quad (11)$$

Since the total energy supplied into a material volume at ξ_i at the boundary surface S is equal to the sum of those entering into the material volume from three perpendicular directions x_i, the magnitude of the applied heat flux Q perpendicular to the surface area at ξ_i (refer to Fig. 1) can be expressed in terms of the average internal heat flux vector by

$$Q(\xi_i) = \bar{q}_j n_j(\xi_i), \quad \text{at } \xi_i \in S \quad (12)$$

with $n_j(\xi_i)$ being the unit normal at ξ_i. Substituting equation (12) into equation (11) and the result into equation (10) then yields

$$f\bar{T}^C_{,i} = \left\{\frac{1}{V}\int_{S^C}\int_S G(x_i, \xi_i)n_j n_i \,\mathrm{d}S\,\mathrm{d}S^C\right\}\bar{q}_j \quad (13)$$

where the unit normal vectors n_j and n_i are functions of space variables ξ_k (on S) and x_k (in V), respectively. If we further denote the quantity enclosed in the braces by H_{ij}, i.e.

$$H_{ij} = \frac{1}{V}\int_{S^C}\int_S G(x_i, \xi_i)n_j n_i \,\mathrm{d}S\,\mathrm{d}S^C \quad (14)$$

equation (8) can be written in the form of

$$\bar{T}_{,i} = \bar{R}_{ij}\bar{q}_j - fR_{ij}\bar{q}^C_j, \quad \text{with } \bar{R}_{ij} = R_{ij} + H_{ij}. \quad (15)$$

In the absence of mesocracks in the solid medium, S^C (and hence H_{ij}) and f are equal to zero and equation (15) reduces to Fourier's law of heat conduction in an averaged form. The influences of the mesocracks on the energy carrying capacity of the solid are implemented in the second term in equation (15) and the matrix H_{ij} defined by equation (14). At this point, determination of the matrix H_{ij} relies on the Green's function $G(x_i, \xi_i)$ and the complicated integral of equation (14). The quantity $\bar{q}^C_j$ in the second term of equation (15) depends on the boundary conditions specified at the mesocrack surfaces as well as the thermodynamic conditions in the aerial closure bounded by the mesocrack surfaces. In the most complicated situation, it can involve the solution of an integral-differential equation due to thermal radiation from the mesocrack surfaces. As a lumped approximation for an *opening* crack with sharp tips and its surface subjected to a constant temperature, the average heat flux $\bar{q}^C_j$ can be expressed in terms of the linear combination of the heat fluxes due to conduction, convection, and radiation:

$$\bar{q}^C_2 = 2[k^C_{2j}\bar{T}^C_{,j} + h(T_0 - T_\infty) + \sigma(T_0^4 - T_\infty^4)],$$
$$\text{and } \bar{q}^C_1 = \bar{q}^C_3 = 0 \quad (16)$$

with T_0 and T_∞ being, respectively, the temperature of the crack surface and the aerial closure, h the heat transfer coefficient of the air, and σ the Stephan–Boltzmann constant of thermal radiation. In equation (16) k^C_{2j} is the conductivity matrix of the air and $\bar{T}^C_{,i}$ the temperature gradient across the aerial closure. The constant 2 in the front is due to the presence of two crack surfaces bounding the aerial closure. For a crack with a slit-geometry and sharp tips, however, we notice that the aerial closure occupies an infinitesimal space between the crack surfaces and its initial temperature T_∞ will be heated up to the level of T_0 after the steady state is reached. These observations suggest a simplified result

$$\bar{q}^C_j = 0, \quad \text{for } j = 1, 2, 3 \quad (17)$$

and according to equation (15)

$$\bar{T}_{,i} = \bar{R}_{ij}\bar{q}_j, \quad \text{with } \bar{R}_{ij} = R_{ij} + H_{ij}. \quad (18)$$

The effects of the mesocracks on the energy-carrying capacity of the solid medium are thus concentrated on the added matrix H_{ij} because the aerial closure is assumed not to carry thermal energy in a steady state. To be also noticed is that equation (18) for the average temperature gradient over the entire body may become exact if an insulation boundary condition, i.e. $q_i \sim T_{,i} = 0$ at S^C, is applied at the crack surfaces. In this case there exists no energy exchange between the air enclosed in the mesocracks and the crack surfaces, and $\bar{q}^C_j$ in equation (15) is equal to zero.

DETERMINATION OF H_{ij}—AN EXAMPLE OF PENNY-SHAPED CRACKS

In this section, the overall (bulk) thermal resistance (or conductivity) of a Fourier solid containing randomly distributed penny-shaped cracks is estimated. To include the interaction effects, the present paper adopts the self-consistent approach [5, 6] which assumes that the material properties of a solid containing a single crack are identical to that of the bulk values.

The effects of the mesocracks on the conductivity matrix of the solid can be obtained if the added matrix H_{ij}, for $i, j = 1, 2, 3$, is determined. Instead of searching for the Green's function $G(x_i, \xi_i)$ and evaluating the complicated integral of equation (14), the matrix H_{ij} can also be determined by combining equations (10) and (13). According to these two equations, H_{ij} can be related to the surface integral around the mesocrack surfaces by

$$\frac{1}{V}\int_{S^C} Tn_i\,\mathrm{d}S^C = \left\{\frac{1}{V}\int_{S^C}\int_S G(x_i,\xi_i)n_j n_i\,\mathrm{d}S\,\mathrm{d}S^C\right\}\bar{q}_j = H_{ij}\bar{q}_j. \quad (19)$$

In applying the self-consistent method, the matrix H_{ij} in equation (19) is calculated for the single crack, and equation (18) is used to estimate the bulk thermal resistance. For a set of the prime coordinate system with the unit normal $\mathbf{e}'_3$ being perpendicular to the penny-shaped crack surface A, as shown by Fig. 2, we may define an equivalent matrix H'_{ij} on the prime coordinates as

$$\int_A [T']n'_i\,\mathrm{d}S = H'_{ij}\bar{q}'_j \quad (20)$$

where $[T']$ denotes the temperature jump, according to the surface integral in equation (19), across the mesocrack surface A the unit normal $\mathbf{n}'$ of which is $\mathbf{e}'_3$. The unit normals $\mathbf{e}'_1$ and $\mathbf{e}'_2$ are coincident with the crack surface and the relationship between $\mathbf{e}'_j$ and the unit normals of the physical coordinate system $\mathbf{e}_i$ is

$$\mathbf{e}_i = \beta_{ij}\mathbf{e}'_j \quad (21)$$

with

$$\beta_{ij} = \begin{bmatrix} -\sin\theta & -\sin\phi\cos\theta & \cos\phi\cos\theta \\ \cos\theta & -\sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi & \sin\phi \end{bmatrix} \quad (22)$$

where $0 \leqslant \theta \leqslant 2\pi$ and $0 \leqslant \phi \leqslant \pi/2$ (see Fig. 2). Determination of H'_{ij} according to equation (20) depends on the temperature jump across the mesocrack surface A. The temperature jump, obviously, depends on the constitutive behavior of the solid medium. In order to illustrate the procedure discussed so far, the temperature field around a penny-shaped crack obtained by Florence and Goodier [15] is used. In their work the three-dimensional Fourier solid is assumed to be isotropic and homogeneous, and the thermal resistance matrix R_{ij} in equation (18) in this case can be

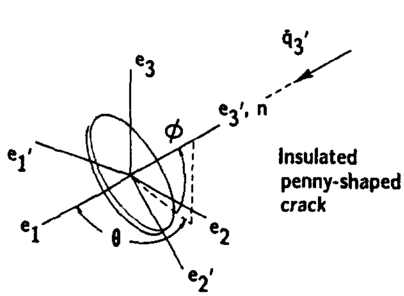

FIG. 2. The relationship between the prime coordinate system attached to the surface of a penny-shaped crack and the physical coordinate system.

expressed as

$$R_{ij} = RI_{ij} \quad \text{for } i,j = 1, 2, \text{and } 3 \quad (23)$$

where I_{ij} is the identity matrix defined as $I_{ij} = \delta_{ij}$ with δ_{ij} being the Kronecker delta function. For the penny-shaped crack subjected to a uniform steady heat flow in the $\mathbf{e}'_3$-direction perpendicular to the crack surfaces, the temperature distributions at the crack surfaces with angular symmetry were derived as

$$T(r) = \pm\frac{2R}{\pi}(a^2-r^2)^{1/2}\bar{q}'_3, \quad \text{for } 0 < r < a \quad (24)$$

where the positive and negative signs correspond, respectively, to those along the upper and lower surfaces of the crack, r measures the radial distance in the plane of the crack from its center, R stands for the thermal resistance of the solid, and a the radius of the penny-shaped crack. Clearly, equation (24) indicates a temperature jump

$$[T'] = \frac{4R}{\pi}(a^2-r^2)^{1/2}\bar{q}'_3 \quad (25)$$

across the crack surfaces. Except for the coefficient $4R/\pi$, we notice that equation (25) presents an identical r-dependency to that in the displacement jump across the crack surfaces [6]. Since $\mathbf{e}'_3$ is the unit normal to the crack surface, substitution from equation (25) into equation (20) yields

$$H'_{33} = \frac{8R}{3}a^3, \quad \text{and } H'_{ij} = 0 \text{ otherwise.} \quad (26)$$

This result shows that the energy-carrying capacity of the solid is only affected by the mesocracks in the direction of the heat flow perpendicular to the crack surfaces.

In applying the self-consistent method, the thermal resistance R in equation (26) is replaced by its bulk value $\bar{R}$, the matrix H'_{ij} obtained in equations (26) in the prime coordinate system is transformed back to the physical coordinates to obtain H_{ij}, and the results are then averaged over all crack orientations to evaluate the bulk resistance. Since the distribution of the mesocracks in the solid is random, these procedures render that

$$H_{ij} = \frac{N}{2\pi}\int_0^{2\pi}\int_0^{\pi/2}\beta_{im}\beta_{jn}H'_{mn}\cos\phi\,\mathrm{d}\phi\,\mathrm{d}\theta \quad (27)$$

with β_{ij} being the transformation matrix defined in equation (22) and hence

$$\begin{aligned}
H_{11} &= H'_{33}I\{\cos^3\phi\cos^2\theta\} \\
H_{12} &= H'_{33}I\{\cos^3\phi\cos\theta\sin\theta\} \\
H_{13} &= H'_{33}I\{\cos^2\phi\sin\phi\cos\theta\} \\
H_{21} &= H_{12}, \quad H_{22} = H'_{33}I\{\cos^3\phi\sin^2\theta\} \\
H_{23} &= H'_{33}I\{\cos^2\phi\sin\phi\sin\theta\} \\
H_{31} &= H_{13}, \quad H_{32} = H_{23} \\
H_{33} &= H'_{33}I\{\sin^2\phi\cos\phi\} \quad (28)
\end{aligned}$$

with H'_{33} being that obtained in equation (26) and the integrator $I\{X\}$ defined as

$$I\{X(\theta,\phi)\} \equiv \frac{N}{2\pi}\int_0^{2\pi}\int_0^{\pi/2} X(\theta,\phi)\,\mathrm{d}\phi\,\mathrm{d}\theta. \quad (29)$$

By noticing the orthogonality of the sine and cosine functions in the domain from 0 to 2π, the components of H_{ij}, for $i, j = 1, 2$, and 3, can be integrated directly according to equations (28) and (29). The result is

$$H_{ij} = \frac{8}{9}\bar{R}fI_{ij} \quad (30)$$

where the total number of cracks in a material volume with a characteristic dimension of a has been substituted by the volume fraction f of the mesocracks [6, 7, 9]. Mathematically, $f = Na^3$. Substituting equation (30) into equation (18) with R_{ij} being the isotropic matrix defined in equation (23), the ratio of the isotropic thermal resistance $\bar{R}/R$ can be solved as

$$\frac{\bar{R}}{R} = \frac{9}{9-8f} \quad (31)$$

and consequently, since the thermal resistance is the reciprocal of the thermal conductivity

$$\frac{\bar{k}}{k} = 1 - \frac{8}{9}f. \quad (32)$$

Equation (32) is identical to that obtained by Hoenig [13] via a different approach employing the electrical and thermal analogy. It presents an expression for the degraded bulk thermal conductivity due to the presence of mesocracks and depends on the mesocrack density f. In a Fourier solid without a crack, f is equal to zero and equation (32) shows no degradation of the bulk thermal conductivity, and hence no change in the energy-carrying process in the solid. In the solid with fully saturated mesocracks, $f = 1$, equation (32) shows that the bulk thermal conductivity will degrade to one-ninth of the intact value. This result must be considered approximate, however, since the validity of the self-consistent approach at high crack density is questionable. Under a temperature gradient established between two material points in the solid, this implies the solid medium full of mesocracks can only carry one-ninth of the thermal energy in comparison with that without a crack. Figure 3 shows that the degradation of the bulk thermal conductivity varies as a function of the mesocrack density. As expected, the amount of degradation linearly increases as the mesocrack density developed in the solid increases.

The results obtained so far are limited to a Fourier solid carrying heat in an isotropic manner. For the solid with anisotropic heat-carrying characteristics, a full matrix will appear for the thermal resistance R_{ij} and the temperature distributions along the upper and lower surfaces of the crack, equation (24), and hence the temperature jump, equation (25), must be rederived. The rest of the procedures stay exactly

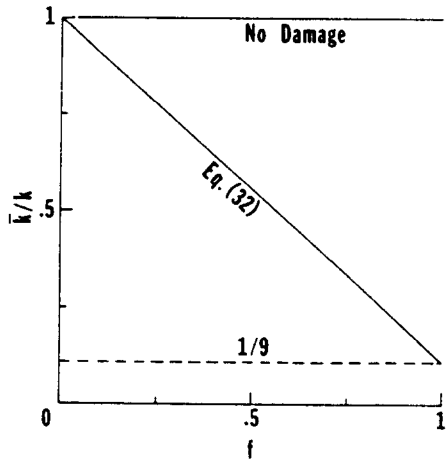

Fig. 3. Degradation of the bulk thermal conductivity as a linear function of the mesocrack density, equation (32).

the same as proceeded so far which will render an anisotropic matrix for H_{ij}. The ratio for the degradation of the thermal conductivity in this case will be much more complicated than that shown in equation (32) and will contain, in general, a total of nine components for $\bar{k}_{ij}$.

GEOMETRY OF DEFECTS

The geometry of defects in a three-dimensional solid is assumed to be randomly distributed penny-shaped cracks in this study. When the geometrical shape of the defects changes, the amount of degradation of the bulk thermal conductivity will change accordingly. This can be seen clearly by inspecting equation (19) where the surface integral on temperature over the entire surface depends on the specific geometry of the defects. It is only for a penny-shaped crack that this integral can be reduced to the temperature jump across the crack surfaces, equation (20).

In making contact with the existing models accounting for the degradation of the bulk thermal conductivity, we compare our results with those obtained by Loeb [16], Budiansky [17], and Agapiou and DeVries [18]. The geometry of inclusions considered in these works are *pores* [16, 18] and *cavities* [17]. In the former case involving pores, the volume fraction refers to the pore fraction cross-sectional area for isometric pores.

In summary, the effects of pore and cavity fractions on the degradation of bulk thermal conductivity are expressed by

$\bar{k}/k = 1 - A_1 f$, Loeb's pore model

$2\bar{k}^2 + [3(f_1k_2 + f_2k_1) - 2(k_1+k_2)]\bar{k} - k_1k_2 = 0$,

Budiansky's cavity model

$\bar{k}/k = (1 - A_2 f)/(1 + A_3 f + A_4 f^2)$,

Agapiou and DeVries' model. (33)

The coefficients A involved in the first and the third equations depend on the material type under consideration. For a material with two constituents, the bulk thermal conductivity in Budiansky's model depends on the respective thermal conductivities (k_1 and k_2) and the volume fractions (f_1 and f_2) of the constituents. Basically, the models proposed by Loeb [16] and the present work vary linearly with respect to the volume fraction of the defects, while those by Budiansky [17] and Agapiou and DeVries [18] are non-linear models. For 304L stainless steel powder metallurgy material, experiments were carried out recently and all the coefficients A in Loeb's and Agapiou and DeVries' models were determined by curve fitting techniques [18]. The results are

$$A_1 = 1.755 \quad \text{in Loeb's model}$$

and

$$A_2 = 1.88, \quad A_3 = 0.38, \quad A_4 = 2.3 \quad \text{in Agapiou and DeVries' model.} \tag{34}$$

The graphical representations for these models are displayed in Fig. 4. Curves 1–4 refer to, respectively, the distributions employing the models proposed by the present analysis, Budiansky, Agapiou and DeVries, and Loeb. The results obtained by Budiansky, Loeb, and Agapiou and DeVries all fall in the threshold of experimental tolerance. The non-linearity of the model proposed by Budiansky seems to be weaker than that by Agapiou and DeVries in the range of $0 < f < 0.4$. As far as the geometry of the defects is concerned, we first notice that the degradation of the thermal conductivity, namely $\bar{k}/k$, is more serious for the case involving randomly distributed pores than that involving mesocracks. The physical interpretation is that under the same value of volume fraction f, the pores reduce the effective heat-carrying material volume more than the mesocracks. In comparing the present model with the linear model proposed by Loeb, we also notice that the slope of

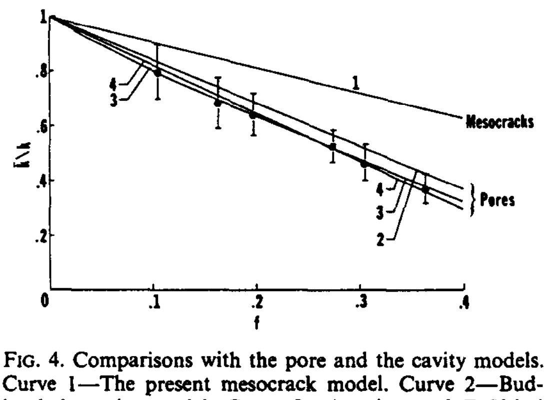

FIG. 4. Comparisons with the pore and the cavity models. Curve 1—The present mesocrack model. Curve 2—Budiansky's cavity model. Curve 3—Agapiou and DeVries' porous model. Curve 4—Loeb's porous model (Curves 3, 4, and the experimental results were obtained by Agapiou and DeVries [18]).

degradation induced by the pores is approximately twice as much as that induced by mesocracks, see equations (32) and (33). This is because the reduction of the material surface carrying the heat flux in the case of pores of average radius R ($4\pi R^2$) is twice as much as that in the case of penny-shaped cracks with the same averaged radius ($2\pi R^2$). The amount of degradation of the bulk thermal conductivity is consequently twice as much. This observation leads to the establishment of a scaling law based on the geometrical shape of defects. Such a trend is currently further examined by considering a three-dimensional solid containing ellipsoidal cavities. Should the rule be valid in general, the ratio of the surface of an ellipsoid to that of a sphere with its diametral plane coincident with one of the major axes of the ellipsoid should give accordingly the ratio of the degradation of the bulk thermal conductivities. This could be a very convenient rule for use in technologies dealing with the heat conduction in porous media.

CONCLUSIONS

The presence of mesocracks in a solid medium reduces the effective material volume in carrying the thermal energy. As a result, the amount of heat carried through the medium should decrease when the mesocrack density increases. An analytical model for evaluating the degradation of the energy-carrying capacity of the solid medium due to mesocracks is proposed in this work which relates the degraded bulk thermal conductivity to the mesocrack density. In a steady state, the degradation of the bulk thermal conductivity for an isotropic Fourier solid is derived in equation (32). It presents a linear relationship between $\bar{k}/k$ and f as shown in Fig. 3. In the absence of the experimental data confirming the proposed analytical model, the observation based on the geometrical shape of the internal cavities validates the theory at least on a qualitative basis. Under the same volume fraction and the characteristic dimension of the internal cavities, as clearly shown in Fig. 4, the degradation of bulk thermal conductivity in a solid due to pores is more serious than that due to mesocracks because the pore geometry reduces the effective material volume carrying the thermal energy more than the penny-shaped crack.

Consideration of the mesocrack damage is especially important in the heat conduction problems with large temperature gradients. The problems in this category involve those with geometrical singularities such as a crack [1, 2, 15] or those with thermal shock discontinuities. In a series of recent studies [19–22] on the thermal shock formation around a rapidly moving heat source or a propagating crack tip, both the temperature and its gradient approach large values in the vicinity of the shock surfaces. In studying the associated modes of material failure in this type of problem, the mesocrack damage could occur prior to the failure modes in a macroscopic level such as yielding and fracture [23, 24], and incorporating the mesocrack damage in estimating the energy-carrying capacity of the material continua is necessary.

Another motivation of the present work is to provide a fundamental basis for the evaluation of the mesocrack damage evolving in the history of a temperature gradient. A damage model established on a continuum level has been proposed by the authors for this purpose [12] which incorporates the *evolution* of the mesocrack density (f) into considerations as the temperature gradient increases. For an isotropic medium, in brief, the degradation of the bulk thermal conductivity was expressed in terms of a damage measure D

$$\frac{\bar{k}}{k} = 1 - D \tag{35}$$

and D depends on the temperature gradient evolving in a specified thermal loading history

$$D = \beta(|\nabla T|/|\nabla T_c|)^n \tag{36}$$

where β and n are the damage parameters depending on the mesostructural integrity of the solid medium, and ∇T_c the threshold value of the critical temperature gradient which, when applied to the solid medium, results in a fully saturated mesocrack distribution ($f = 1$) in the solid. In such a model, the temperature gradient is attributed as the driven force for the formation of mesocracks in the thermal loading history. In bridging this model with the one being developed presently, we see immediately that

$$\frac{8f}{9} = \beta(|\nabla T|/|\nabla T_c|)^n. \tag{37}$$

When the temperature gradient $|\nabla T|$ reaches the threshold value of $|\nabla T_c|$, the mesocrack density f reaches a value of one and equation (37) yields a result of

$$\beta = 8/9 \simeq 0.89. \tag{38}$$

A detailed study on the constitutive behavior of the Fourier solid with mesocrack damage for β ranging from 0 to 1.5 has already been made in the earlier work [12]. For $\beta = 0.89$ as obtained in equation (38), a softening response between the heat flux and the temperature gradient will exist in the post-peak regime. Post-peak softening is a distinct feature due to the evolution of the mesocracks in the solid which depicts that when the temperature gradient established in a material volume exceeds a critical value, the mesocrack density thereby dramatically increases with the temperature gradient and the heat flux through the medium consequently decreases even if the temperature gradient further increases. The value of β obtained in equation (38) is left for the future experiments to verify.

The evolution of the mesocrack density in the history of the temperature gradient from 0 to $|\nabla T_c|$, refer to equation (37), also imposes an additional complexity in the analysis for the heat conduction in the solid with damage. For example, because the mesocrack density developed in a material volume depends on the temperature gradient established at a certain step in a specified path of temperature loading, it essentially implies the need of decomposing the final temperature level imposed on the body into several increments. At the end of a temperature increment, the mesocrack density is calculated according to the established temperature gradient, and the resulting bulk thermal conductivity is calculated according to equation (35) which is then used in the next temperature increment for calculating the corresponding thermal response. This procedure should be repeated until the temperature level reaches the final value and the model suggests the accumulation of the mesocrack damage in an incremental manner. To be noticed is the path-dependency of the damage evolution, and hence the thermal field, proposed in this model. In the earlier work done by the authors [12], both the path-dependent and path-independent approaches were studied and detailed discussions were made to stress the physical contents of the damage evolution in the thermal loading history.

In extending the present model to account for the mesocrack damage in unsteady problems, the difficulty lies in the estimation of $\bar{q}_j^C$ in equation (15). This term deals with the heat transfer in the aerial closure between the mesocrack surfaces. In addition to the modelling problems for the heat transfer modes in the closure, we also have to determine consistently the temperature distributions on the crack surfaces according to the same boundary conditions as those used in estimating the average heat flux $\bar{q}_j^C$. Due to the complexity of the crack problems involving the solutions of dual integral equations [15, 25, 26], a closed form solution for complicated boundary conditions is difficult to obtain and a numerical algorithm to evaluate the integrator in equation (29) may have to be used. The problem considered in this paper is a simple case possessing a closed form solution for the temperature jump and it serves the purpose of illustrating the concept of mesocrack damage in the process of heat transport.

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DEGRADATION GLOBALE DES SOLIDES CONDUCTEURS PAR DES MESOCRAQUELURES

Résumé—Un modèle continu de la conduction thermique est proposé pour tenir compte de la dégradation de la conductivité globale due à la formation de défauts à petite échelle dans le solide. Spécifiquement, des défauts en forme de craquelure, ou mésocraquelure sont considérés. Une théorie de champ est analytiquement développée qui isole les effets de mésocraquelures dans une matrice de résistance thermique. Le concept de dommage par mésocraquelure dans le mécanisme de transfert thermique est illustré en considérant un milieu tridimensional et isotrope obéissant à la loi de Fourier. On trouve que l'importance de la dégradation de la conductivité thermique augmente linéairement avec la densité de mésocraquelure. Dans le cas où le solide est saturé de mésocraquelures, on obtient une valeur limite de la conductivité thermique $\bar{k}$ qui est le neuvième de la valeur intacte. Une discussion détaillée sur la valeur limite est faite qui porte sur l'identification des paramètres de dommage dans le modèle proposé ultérieurement, à gradient de température, pour tenir compte du dommage par microcraquelure.

VERMINDERUNG DER WÄRMELEITFÄHIGKEIT VON FESTKÖRPERN DURCH RISSE MITTLERER GRÖSSE

Zusammenfassung—In dieser Arbeit wird ein Modell der Wärmeleitung in Festkörpern vorgestellt, das die Verminderung der makroskopischen Wärmeleitfähigkeit durch die Bildung von Mikrofehlern berücksichtigt. Insbesondere werden Fehler in Form von Rissen oder Meso-Rissen betrachtet. Es wird auf analytische Weise eine Feldtheorie entwickelt, die den Einfluß der Meso-Risse in der Matrix der thermischen Widerstände separiert. Gefolgt von einer allgemeineren Formulierung wird zunächst das Konzept der Meso-Rißentstehung bei Wärmetransportvorgängen erläutert. Hierzu wird ähnlich wie bei Fourier's Wärmeleitgesetz ein dreidimensionales isotropes, wärmetransportierendes Medium betrachtet. Es zeigt sich, daß die Verminderung der makroskopischen Wärmeleitfähigkeit linear mit der Dichte von Meso-Rissen zunimmt. Wenn der Festkörper mit Meso-Rissen gesättigt ist, ergibt sich ein Grenzwert der Wärmeleitfähigkeit, der nur ein Neuntel des Ausgangswertes beträgt. Es folgt eine detaillierte Diskussion dieses Grenzwertes. Dabei wird besonderer Wert auf die Identifikation des Schadensparameters in dem Temperaturgradienten-Modell gelegt, das bereits früher für die Betrachtung der Meso-Riß-Schädigung unter einem anderen Gesichtswinkel vorgeschlagen worden ist.

ТЕПЛОПРОВОДНОСТЬ ТВЕРДЫХ ТЕЛ С МЕЗОТРЕЩИНАМИ

Аннотация—Предложена эффективная модель теплопроводности в среде с повреждениями, учитывающая уменьшение теплопроводности вследствие образования микромасштабных дефектов в твердом теле. Рассмотрены дефекты в форме трещин или мезотрещин. Влияние повреждений за счет мезотрещин изучено в рамках общей теории теплопроводности на основе закона фурье. Найдено, что степень ухудшения теплопроводности линейно возрастает с увеличением плотности мезотрещин. Для случая, когда твердое тело насыщено мезотрещинами, получено предельное значение коэффиента тепцопроводности $\bar{k}$, составляющее 1/9 от значения этой величины для неповрежденного тела. При обсуждении этого предельного значения особое внимание уделятся идентификации параметров повреждения в данной и ранее предложенных моделях.